Star Edge-Coloring

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A *star edge-coloring* of a graph is a proper edge-coloring with forbidden bichromatic paths and cycles of length 4. The least number of colors that suffice for an edge-coloring of a graph G is called the *star chromatic index*, denoted $\chi'_{\rm st}(G)$. Motivated by the vertex analogue, the notion has been introduced by Liu and Deng [2], who proved the first upper bound on the star chromatic index of any graph with maximum degree at least 7.

Currently the best upper bound for general graphs has been established by Dvořák, Mohar, and Šámal [1] using their upper bound on the star chromatic index of complete graphs. They showed that for every $\varepsilon > 0$ there exists a constant C such that

$$\chi'_{\rm st}(K_n) \leq C\ n^{1+\varepsilon},$$

for every positive n. They asked if the star chromatic index of complete graphs K_n is linear in terms of n. The answer to this question is still not known.

Apart from the above, in [1], the authors also considered subcubic graphs, proved that for any subcubic graph 7 colors suffice for a star edge-coloring, conjectured that the bound can be decreased to 6, and asked about the bounds for the list version of star edge-coloring. All these questions initiated an intense (and still ongoing) research on the topic. In the talk, we will give a survey of known results, present the most interesting proofs and techniques, and propose additional possible directions of research on the topic.

References

[1] Z. Dvořák, B. Mohar, R. Šámal: Star chromatic index, J. Graph Theory 72 (2013) 313–326.

[2] X. S. Liu, K. Deng: An upper bound on the star chromatic index of graphs with $\Delta \geq 7$, J. Lanzhou Univ. (Nat. Sci.) 44 (2008) 94–95.