

New Approximation algorithm for $(1, 2)$ -TSP in Semi-Streaming model

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Joint work with Tomáš Kaiser †

Abstract

The *graph streaming model* is a model of computation that encapsulates the requirement of processing large graphs in parts by keeping a significantly small portion of it in the available memory at any point in time. Among all the variants of graph streaming models the most popular one is *semi-streaming* model, introduced by Feigenbaum et al. [3]. In this model of computation the graph edges of a graph G on n vertices arrive as a data stream (as the name suggests) while the user has a limited memory of $O(n \cdot \text{polylog}(n))$ and is allowed to make a constant number of *passes* over the stream. The problems that are known to be polynomial time solvable in classical computational model become difficult in this restricted setup, e.g. maximum matching. We study a variant of Traveling Salesman Problem (TSP), known as $(1, 2)$ -TSP, where the edge weights of the given graph are either 1 or 2, in semi-streaming model. Although, $(1, 2)$ -TSP may appear to be an artificial problem, it is in fact the natural next step after the graphic TSP for gaining deeper insight into the realm of metric TSP. We are interested in an approximate solution to the problem in semi-streaming model as the problem is NP-Hard. To achieve the goal, we seek an approximate solution to an underlying problem known as *Maximum Path Cover (MPC)* problem. Given a graph, MPC asks for a collection of vertex disjoint paths such that the number of edges covered by these paths is maximized. This problem is equivalent to the general *path cover* problem where the objective is to minimize the number of vertex disjoint paths that cover all the vertices of a given graph. Although both $(1, 2)$ -TSP and MPC are extensively studied in the literature for classical computational model, there has been very few recent results for the problems in semi-streaming model. In this talk, we will look into our approximation algorithm for MPC that improves the result of Alipour et al. [1] of $(\frac{2}{3} - \epsilon)$ -approximation algorithm in $\text{poly}(\frac{1}{\epsilon})$ passes for MPC which is an improvement over the previous $\frac{1}{2}$ -approximation by Behnezhad et al. [2]. Alipour et al. [1] show a $(\frac{4}{3} + \epsilon)$ -approximation algorithm in $\text{poly}(\frac{1}{\epsilon})$ passes for $(1, 2)$ -TSP as an immediate consequence of their MPC result, improving upon the previous $\frac{3}{2}$ -approximation factor algorithm by Behnezhad et al. [2]. Similarly, our MPC result improves upon the existing approximation ratio for $(1, 2)$ -TSP as a corollary. We also show a family of tight examples which limits the proposed algorithm for future work in [1] (Algorithm 4 in Section 6) to provide a better than $(\frac{2}{3} - \epsilon)$ -approximation for MPC in Semi-streaming model, settling one of the open problems in [1].

References

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