Is the birank function of a delta-matroid the rank function of a matroid?

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In this short talk, we present an ongoing project on delta-matroids in the context of enveloping matroids. Matroid theory was independently established by Hassler Whitney and Takeo Nakasawa. Whitney's motivation was to provide an abstraction of "independence" that is common to both graphs and matrices. Robust developments in the theory have since taken place beyond these two areas. Delta-matroids were introduced by André Bouchet to study combinatorial objects for which the greedy algorithm returns an optimal solution. They were later applied to study many other problems, including Eulerian tours in graphs and graph embeddings.

Delta-matroids are a generalisation of matroids. A recent development has shed new light on their connection in geometry. Through the lens of covering of set systems or polytopes, it is natural to study whether a delta-matroid admits an enveloping matroid. If so, one way is to extend a rank function of a delta-matroid to a rank function of a matroid. Our work focuses on understanding and generalising forbidden conditions related to the extension. This is a joint work with Relinde Jurrius, Dmitry Mineev and Lauren Nowak.