Max Path Cover in Semi-streaming model

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Joint work with Tomáš Kaiser

Abstract

The *graph streaming model* is a relatively new model of computation that encapsulates the requirement of processing large graphs in parts by keeping a significantly small portion of it in the available memory at any point in time. Among all the variants of graph streaming models the most popular one is *semi-streaming* model, introduced by Feigenbaum et al. [3]. In this model of computation the graph edges of a graph G on n vertices arrive as a data stream (as the name suggests) while the user has a limited memory of $O(n \cdot \text{polylog}(n))$ and is allowed to make a constant number of *passes* over the stream. The problems that are known to be polynomial time solvable in classical computational model become difficult in this restricted setup, e.g. maximum matching. We study *Maximum Path Cover (MPC)* problem in the semi-streaming setup. Given a graph, MPC asks for a collection of vertex disjoint paths such that the number of edges covered by these paths is maximized. This problem is equivalent to the general *path cover* problem where the objective is to minimize the number of paths that cover all the vertices of a given graph. In this talk, we will focus on a paper of Alipour et al. [1] where they show a $(\frac{2}{3} - \epsilon)$-approximation algorithm in $\text{poly}(\frac{1}{\epsilon})$ passes for MPC which is an improvement over previous $\frac{1}{2}$-approximation by Behnezhad et al. [2]. MPC has a close connection with a variant of Traveling Salesman Problem (TSP), known as $(1, 2)$-TSP, where the edge weights of the given graph are either 1 or 2. Although, $(1, 2)$-TSP may appear to be an artificial problem, it is in fact the natural next step after the graphic TSP for gaining deeper insight into the realm of metric TSP. Alipour et al. [1] show a $(\frac{4}{3} + \epsilon)$-approximation algorithm in $\text{poly}(\frac{1}{\epsilon})$ passes for $(1, 2)$-TSP as an immediate consequence of their MPC result, improving upon the previous $\frac{3}{2}$-approximation factor algorithm by Behnezhad et al. [2]. We will also look into the limitations of their method and a possible direction towards a better approximation algorithm for MPC in semi-streaming setup which is our work in progress.

References

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